Inference for Group Interaction Experiments

Jiawei Fu (Yale ISPS)
Cyrus Samii (NYU Politics)
Ye Wang (UNC Political Science)

September 5, 2024

Does Descriptive Representation Facilitate Women's Distinctive Voice? How Gender Composition and Decision Rules Affect Deliberation

Tali Mendelberg Princeton University
Christopher F. Karpowitz Brigham Young University
Nicholas Goedert Washington University in St. Louis

(Mendelberg et al. 2013, *AJPS*).

Introduction

Table 1 Experimental Conditions and Sample Size

	# Unanimous Groups	# Majority Groups	Total # Groups	# of Individuals
0 Females	8	7	15	75
1 Female	10	9	19	95
2 Females	6	7	13	65
3 Females	9	7	16	80
4 Females	8	8	16	80
5 Females	7	8	15	75
Total # of Groups	48	46	94	
# of Individuals	240	230		470

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- Outcome: women's mentions of care issues.

For multivariate tests, we employ probit for *Mention* and OLS for *Frequency*.[12] The unit of analysis is the individual speaker, and we employ cluster robust standard errors to account for the fact that individuals are nested within groups. We estimate two models. For predicted

Motivation

Other Examples

- Teacher/classroom experiments.
- Group deliberation experiments.
- Game theoretic lab experiments.
- Group norm experiments.
- Intergroup contact experiments.

Common features

- People put into groups to interact.
- Group level factors affect interactions.
- Interested in effects on individuals' outcomes.

Not addressed by existing literature

- Interference w/ fixed groups (Hudges & Halloran 2008; Tchetgen-Tchetgen & VanderWeele 2012).
- Group-aggregate analysis (Li et al. 2019).
- Permutation-based inference (Basse et al. 2024).
- Cluster randomization without interference (Su & Ding 2021; Abadie et al. 2023; Bugni et al. 2024).

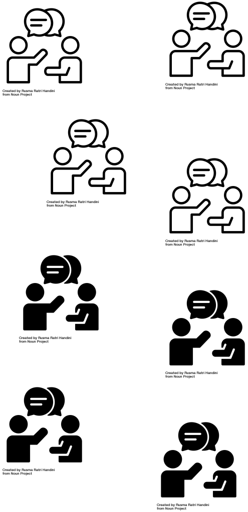
Approach

- Robust inference for causal effects under minimal DGP restrictions.
- Design-based inference with randomization and sampling from super-population.
- Understand implications of interference and group formation.
- Analyze common practices:
  - Individual level analysis with diff-in-means/regression
  - Cluster-robust inference.

Approach

Design:

- Reference population $\mathcal{U}$ with $U := |\mathcal{U}|$.
- Superpopulation: $|\mathcal{U}| \to \infty$.
- Case 0: Groups are fixed.
  - Randomly sample G groups.
  - Groups are size M each, and $N = MG$
- Case 1: Randomly assign units to groups.
  - Randomly sample N units.
  - Partition into G groups of size M, with $N = MG$.
- Group index $A_i \in \{1, ..., G\}$ ordered such that groups $g = 1, ..., G_1$ treated, $g = G_1 + 1, ..., G$ control.
- Group treatment $Z_g \in \{0, 1\}$.
- Units in group g are $\mathcal{A}(g)$
- (N_t, N_c) number of units in treatment, control.



Approach

- $M = 3$, $G = 2$, and so $N = 6$.
- For Case 1: potential group partitions:

[[1]] (1,2,6)(3,4,5)	[[6]] (1,3,5)(2,4,6)
[[2]] (1,2,5)(3,4,6)	[[7]] (1,3,4)(2,5,6)
[[3]] (1,2,3)(4,5,6)	[[8]] (1,4,6)(2,3,5)
[[4]] (1,2,4)(3,5,6)	[[9]] (1,4,5)(2,3,6)
[[5]] (1,3,6)(2,4,5)	[[10]] (1,5,6)(2,3,4)

- Potential outcomes half-matrix:

i	Y_i(P1)	Y_i(P2)	Y_i(P3)	Y_i(P4)	Y_i(P5)	Y_i(P6)	Y_i(P7)	Y_i(P8)	Y_i(P9)	Y_i(P10)
1	X	X	X	X	X	X	X	X	X	X
2	X	X	X	X	.	.	.	.	.	.
3	.	.	X	.	X	X	X	.	.	.
4	.	.	.	X	.	.	X	X	X	.
5	.	X	.	.	.	X	.	.	X	X
6	X	.	.	.	X	.	.	X	.	X

Approach

Potential outcomes:

- Regularity assumptions: bounded outcomes.
- Case 0: No interference
  - $Y_i(\mathbf{z}, \mathbf{a}) = Y_i(z_{a_i})$.
  - $Y_i = \sum_{z \in \{0,1\}} Y_i(z)\mathbf{I}(Z_{A_i} = z)$.
- Case 1: Partial interference
  - $Y_i(\mathbf{z}, \mathbf{a}) = Y_i(z_{a_i}, \mathcal{A}(a_i))$.
  - $Y_i = \sum_{\omega \in \overline{\mathcal{A}}_i} \sum_{z \in \{0,1\}} Y_i(z, \{i, \omega\})\mathbf{I}(Z_{A_i} = z)\mathbf{I}(\mathcal{A}(A_i) = \{i, \omega\})$,
    where $\overline{\mathcal{A}}_i$ is set of possible group partners for i.

Approach

		Group formation	
		No	Yes
Interference	No	✍	✍
	Yes	✍	✍

Estimators and estimands:

- Consider unit-level difference in means: $\hat{\tau} = \overline{Y}_1 - \overline{Y}_0$.
  - (Equiv. to group level difference when M is constant).
  - With fixed groups, targets unit-level ATE.
  - With random group assignment or interference?
- Consider "cluster-robust" inference using CR0 (Liang & Zeger 1986).
  - With fixed groups and no interference, SP-consistent for $\text{Var}\left(\hat{\tau}\right)$.
  - With random group assignment or interference?
  - SP consistency analysis uses ANOVA decomposition $\text{Var}\left[\hat{\tau}\right] = \mathbb{E}[\text{Var}_D[\hat{\tau}|S]] + \text{Var}\left[\mathbb{E}_D[\hat{\tau}|S]\right]$.

Results

Result 1: SP-unbiasedness of $\hat{\tau}$ for Average Treatment Effect (ATE), Average Total Effect (AToE), or Average Marginalized Effect (AME).

- No GF, No Interference: ATE
- Random GF, No Interference: ATE.
- No GF, Interference: AToE.
- Random GF and Interference: AME defined as

$$\mathsf{E}_{\mathcal{N},\mathbf{A}}[\hat{\tau}] = \mathsf{E}_{\mathcal{N}} \left\{ \frac{1}{N} \sum_{i=1}^{N} \frac{1}{|\overline{\mathcal{A}}_i|} \sum_{\omega \in \overline{\mathcal{A}}_i} [Y_i(1, \{i, \omega\}) - Y_i(0, \{i, \omega\})] \right\}$$

Average of individual marginal effects, marginalizing over partners.

Results

- For randomized groups, recall group partitions:

[[1]]	(1,2,6)(3,4,5)	[[6]]	(1,3,5)(2,4,6)
[[2]]	(1,2,5)(3,4,6)	[[7]]	(1,3,4)(2,5,6)
[[3]]	(1,2,3)(4,5,6)	[[8]]	(1,4,6)(2,3,5)
[[4]]	(1,2,4)(3,5,6)	[[9]]	(1,4,5)(2,3,6)
[[5]]	(1,3,6)(2,4,5)	[[10]]	(1,5,6)(2,3,4)

- Potential outcomes half-matrix:

i	Y_i(P1)	Y_i(P2)	Y_i(P3)	Y_i(P4)	Y_i(P5)	Y_i(P6)	Y_i(P7)	Y_i(P8)	Y_i(P9)	Y_i(P10)
1	X	X	X	X	X	X	X	X	X	X
2	X	X	X	X	.	.	.	.	.	.
3	.	.	X	.	X	X	X	.	.	.
4	.	.	.	X	.	.	X	X	X	.
5	.	X	.	.	.	X	.	.	X	X
6	X	.	.	.	X	.	.	X	.	X

- Estimand defined by row means. Estimator defined by column means.
- Expectation is row mean of column means.

Results

Result 2: Cluster-robust "CR0" estimator decomposition.

$$\widehat{\text{Var}}\,(\hat{\beta}) = \frac{1}{N_t}\frac{\sum_{g=1}^{K}(\sum_{i=1}^{M}\hat{u}_{gi})^2}{N_t} + \frac{1}{N-N_t}\frac{\sum_{g=K+1}^{G}(\sum_{i=1}^{M}\hat{u}_{gi})^2}{N-N_t}$$

We have:

$$\frac{1}{N_t}\frac{\sum_{g=1}^{K}(\sum_{i=1}^{M}\hat{u}_{gi})^2}{N_t} = \frac{1}{(N_t)^2}\{(1-\frac{M}{N_t})\sum_{g\in G^T}[\sum_{i\in\mathcal{A}(g)} Y_{gi}^2 + \sum_{i\neq j} Y_{gi}Y_{gj}] - (\frac{2M}{N})\sum_{g\in G^T}\sum_{i\in\mathcal{A}(g),j\in\mathcal{A}(g')} Y_iY_j\}$$

and similar for the control group term.

Results

Result 3: SP-consistency of "CR0" with no group formation and no interference.

$$
\begin{aligned}
\text{Var}\,[\hat{\tau}] &= \mathbb{E}[\text{Var}_D[\hat{\tau}|S]] + \text{Var}\,[\tau_S] \\
&= \frac{2}{N}[\frac{1}{U}\sum_{i=1}^{U} Y_i^2(1)] + \frac{2}{N}[\frac{1}{U}\sum_{\substack{i\neq j \\ \{j,\overline{\mathcal{A}}_j\}=\{i,\overline{\mathcal{A}}_i\}}} Y_i(1)Y_j(1)] - \frac{2M}{N}[\frac{1}{U(U-M)}\sum_{\substack{i\neq j \\ \{j,\overline{\mathcal{A}}_j\}\cap\{i,\overline{\mathcal{A}}_i\}=\emptyset}} Y_i(1)Y_j(1)] \\
&\quad + \frac{2}{N}[\frac{1}{U}\sum_{i=1}^{U} Y_i^2(0)] + \frac{2}{N}[\frac{1}{U}\sum_{\substack{i\neq j \\ \{j,\overline{\mathcal{A}}_j\}=\{i,\overline{\mathcal{A}}_i\}}} Y_i(0)Y_j(0)] - \frac{2M}{N}[\frac{1}{U(U-M)}\sum_{\substack{i\neq j \\ \{j,\overline{\mathcal{A}}_j\}\cap\{i,\overline{\mathcal{A}}_i\}=\emptyset}} Y_i(0)Y_j(0)]
\end{aligned}
$$

Unbiased analogue estimator:

$$
\begin{aligned}
\widehat{\text{Var}}\,[\hat{\tau}] &= \frac{1}{(N_t)^2}\{\sum_{g\in G^T}[\sum_{i\in\mathcal{A}(g)} Y_{gi}^2 + \sum_{i\neq j\in\mathcal{A}(g)} Y_{gi}Y_{gj}] - \frac{2M}{N-2M}\sum_{g\in G^T}\sum_{i\in\mathcal{A}(g),j\in\mathcal{A}(g')} Y_{gi}Y_{gj}\} \\
&\quad + \frac{1}{(N_c)^2}\{\sum_{g\in G^C}[\sum_{i\in\mathcal{A}(g)} Y_{gi}^2 + \sum_{i\neq j\in\mathcal{A}(g)} Y_{gi}Y_{gj}] - \frac{2M}{N-2M}\sum_{g\in G^C}\sum_{i\in\mathcal{A}(g),j\in\mathcal{A}(g')} Y_{gi}Y_{gj}\}
\end{aligned}
$$

Results

Result 4: SP-consistency of "HC0" with group formation but no interference.

Unbiased analogue estimator:

$$\begin{aligned}
\widehat{\text{Var}}[\hat{\tau}] &= \frac{1}{(N_t)^2}\{\sum_{g\in G^T}[\sum_{i\in\mathcal{A}(g)} Y_{gi}^2 - \frac{2}{N-2}\sum_{i\neq j\in\mathcal{A}(g)} Y_{gi}Y_{gj}] - \frac{2}{N-2}\sum_{g\in G^T}\sum_{i\in\mathcal{A}(g),j\in\mathcal{A}(g')} Y_{gi}Y_{gj}\} \\
&+ \frac{1}{(N_c)^2}\{\sum_{g\in G^C}[\sum_{i\in\mathcal{A}(g)} Y_{gi}^2 - \frac{2}{N-2}\sum_{i\neq j\in\mathcal{A}(g)} Y_{gi}Y_{gj}] - \frac{2}{N-2}\sum_{g\in G^C}\sum_{i\in\mathcal{A}(g),j\in\mathcal{A}(g')} Y_{gi}Y_{gj}\} \\
&= \frac{1}{N_t}[\frac{1}{N_t-1}\sum_{i\in T}(Y_i - \overline{Y_i^t})^2] + \frac{1}{N_c}[\frac{1}{N_c-1}\sum_{i\in C}(Y_i - \overline{Y_i^c})^2].
\end{aligned}$$

Resembles an individualized randomized experiment.

Results

Result 5: SP-consistency of "CR0" with no group formation but with interference.

Under SP, variance has the same structure as the no group formation, no interference case, since groups are fixed and so group potential outcomes profiles (due to spillover or peer effects) are fixed.

Results

Result 6: SP-consistency of "CR0" with group formation and interference.

After much tedious algebra, the SP-variance turns out to be:

$$
\begin{aligned}
\mathsf{Var}\left[\hat{\tau}\right] = \frac{4}{N^2}\frac{N\pi_{sp}}{2U}\{ & \sum_{i=1}^{U}\sum_{\omega\in\overline{A}_i} Y_i^2(1,\{i,\omega\}) + \sum_{i\neq j}\sum_{\omega\in\tilde{\mathcal{A}}_{ij}} Y_i(1,\{i,j,\omega\})Y_j(1,\{i,j,\omega\}) \\
& -(\frac{N\pi_{SP}}{2U}-p_{sp})\sum_{i\neq j}\sum_{\omega,\omega':\{i,\omega\}\cap\{j,\omega'\}=\emptyset} Y_i(1,\{i,\omega\})Y_j(1,\{j,\omega'\})\} \\
& +\frac{4}{N^2}\frac{N\pi_{sp}}{2U}\{\sum_{i=1}^{U}\sum_{\omega\in\overline{A}_i} Y_i^2(0,\{i,\omega\}) + \sum_{i\neq j}\sum_{\omega\in\tilde{\mathcal{A}}_{ij}} Y_i(0,\{i,j,\omega\})Y_j(0,\{i,j,\omega\}) \\
& -(\frac{N\pi_{SP}}{2U}-p_{sp})\sum_{i\neq j}\sum_{\omega,\omega':\{i,\omega\}\cap\{j,\omega'\}=\emptyset} Y_i(0,\{i,\omega\})Y_j(0,\{j,\omega'\})\}
\end{aligned}
$$

HC0 converges to same limit.

Results

Summary of super-population inference results:

		Group formation	
		No	Yes
Interference	No	$\hat{\tau}$ Targets ATE CR0 SP-Cons.	$\hat{\tau}$ Targets ATE HC0 SP-Cons.
	Yes	$\hat{\tau}$ Targets ATotalE CR0 SP-Cons.	$\hat{\tau}$ Targets AMarginalizedE CR0 SP-Cons.

Upshot: cluster robust always "works" for SP inference, although targeting different estimands and maybe inefficient if no interference.

Results

Simulation study

- $U = 400,000$, $N = 100, 200, 400$, $M = 4$.
- Untreated potential outcomes a function of individual and group member covariates.
- When interference is present, treatment effects are a function of others' covariates too.
- 1,000 simulation runs.

Results

	tau	\hat tau	True Var.	HC2/Truth	CR0/Truth	Analogue/Truth
N=800						
SP, SUTVA, FG	0.92	0.93	0.0062	0.59	1.01	1.03
SP, No SUTVA, FG	1.93	1.93	0.0165	0.39	0.98	1.00
SP, SUTVA, RG	0.92	0.92	0.0037	0.98	0.96	0.98
SP, No SUTVA, RG	2.12	2.12	0.0155	0.44	0.98	1.00
N=200						
SP, SUTVA, FG	0.92	0.93	0.0128	0.57	0.96	1.00
SP, No SUTVA, FG	1.44	1.44	0.0208	0.45	0.95	0.99
SP, SUTVA, RG	0.93	0.92	0.0069	0.99	0.96	0.99
SP, No SUTVA, RG	1.93	1.93	0.0267	0.43	0.93	0.97
N=100						
SP, SUTVA, FG	0.93	0.92	0.0264	0.57	0.91	0.99
SP, No SUTVA, FG	2.07	2.06	0.0830	0.35	0.91	1.00
SP, SUTVA, RG	0.92	0.92	0.0156	0.99	0.91	0.99
SP, No SUTVA, RG	2.09	2.10	0.0611	0.43	0.94	1.02
N=80						
SP, SUTVA, FG	0.92	0.93	0.0320	0.56	0.90	1.00
SP, No SUTVA, FG	1.99	1.98	0.0927	0.35	0.88	0.97
SP, SUTVA, RG	0.92	0.92	0.0164	1.04	0.93	1.04
SP, No SUTVA, RG	1.63	1.62	0.0414	0.54	0.91	1.01

Results

What about finite population?

- All cases contain cross-potential outcome product terms analogous to term that biases the Neyman variance for SATE inference.
- With group formation and interference, in addition, there are cross-world terms for groups that that can never be observed together.
- Generally speaking the SP inference will be conservative.

Conclusion

- Group interaction experiments are a very common design.
- But not covered by existing results, particularly when we want individual-level analysis.
- Approach is to use design-based, *super population* inference.
- Yields justifications for conventional approaches and robust interpretation.